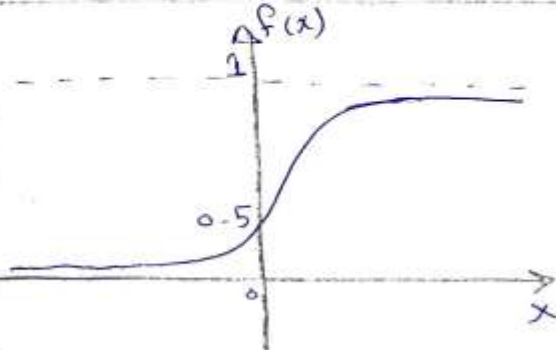
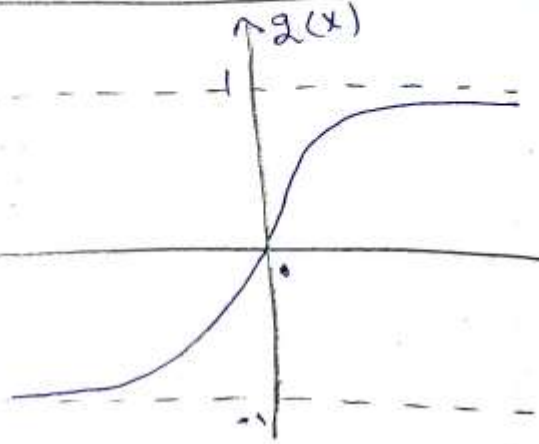
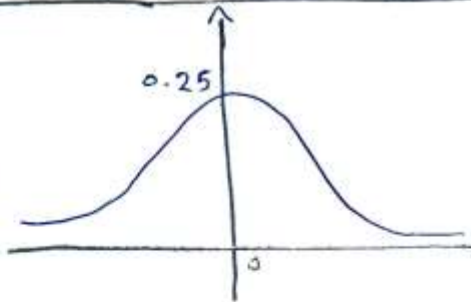
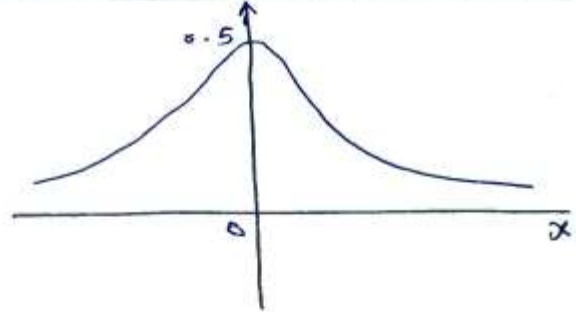


Lec 1 · Neural network

→ A comparison between binary and bipolar sigmoidal functions.

	Binary sigmoid	Bipolar sigmoid
Definition	$f(x) = \frac{1}{1 + e^{-x}}$	$g(x) = \frac{2}{1 + e^{-x}} - 1$ $= \frac{1 - e^{-x}}{1 + e^{-x}}$
Graph		
Range	$0 < f(x) < 1$	$-1 < g(x) < 1$
Relation between $f(x)$, $g(x)$	$g(x) = 2f(x) - 1$	
Value of x	$x = \ln \frac{f(x)}{1 - f(x)}$	$x = \ln \left[\frac{1 + g(x)}{1 - g(x)} \right]$

Derivative	$\frac{dP(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$ $= P(x) [1 - P(x)]$	$\frac{dQ(x)}{dx} = \frac{2e^{-x}}{(1+e^{-x})^2}$ $= 0.5 (1 - Q^2(x))$
Graph of derivative		
Range of values of derivative	$0 < \frac{dP(x)}{dx} \leq 0.25$	$0 < \frac{dQ(x)}{dx} \leq 0.5$
Relation between $\frac{dQ(x)}{dx}$, $\frac{dP(x)}{dx}$	$\frac{dQ(x)}{dx} = 2 \frac{dP(x)}{dx}$	

Sigmoidal Functions of the forms:

$$① f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$② g(x) = \frac{2}{1 + e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$③ h(x) = \tanh(\alpha x)$$

where α is +ve Parameter

$$a) f(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that $x = \frac{1}{\alpha} \ln \frac{f(x)}{1-f(x)}$

القانون ده موجود فيا قبل لى α كانت تساوى 1

Proof

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$f(x) + e^{-\alpha x} f(x) = 1$$

$$e^{-\alpha x} = \frac{1 - F(x)}{F(x)}$$

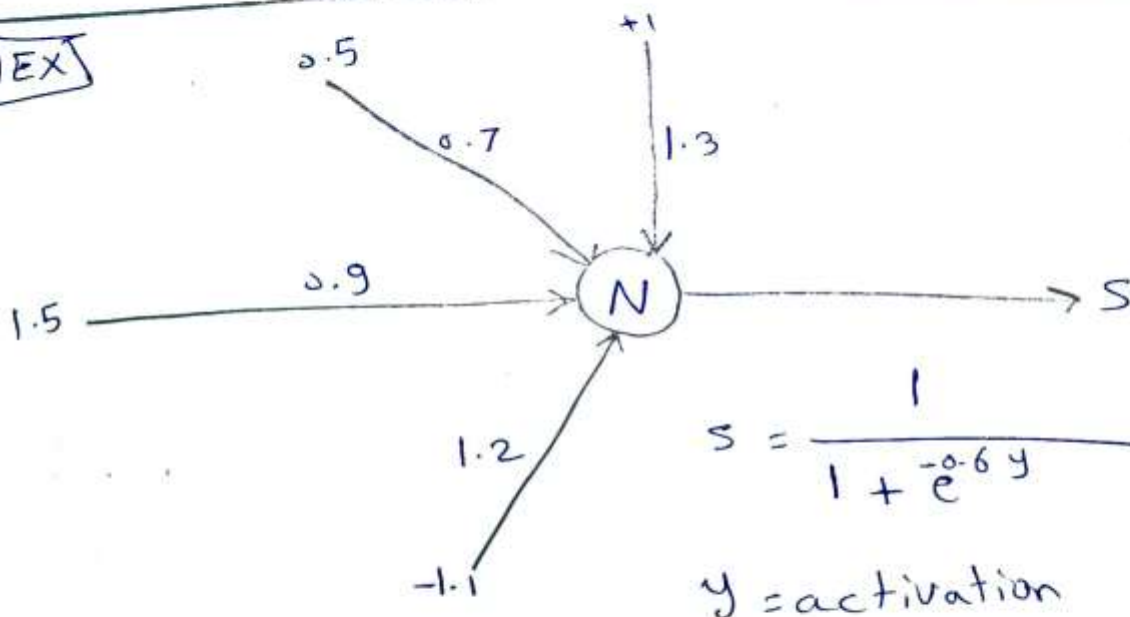
ناتج L_n للفرقة

$$-\alpha x = \ln \frac{1 - F(x)}{F(x)}$$

$$\alpha x = \ln \frac{F(x)}{1 - F(x)} \Rightarrow x = \frac{1}{\alpha} \ln \frac{F(x)}{1 - F(x)}$$

$$\alpha = \frac{1}{x} \ln \frac{F(x)}{1 - F(x)} \quad \#$$

EX

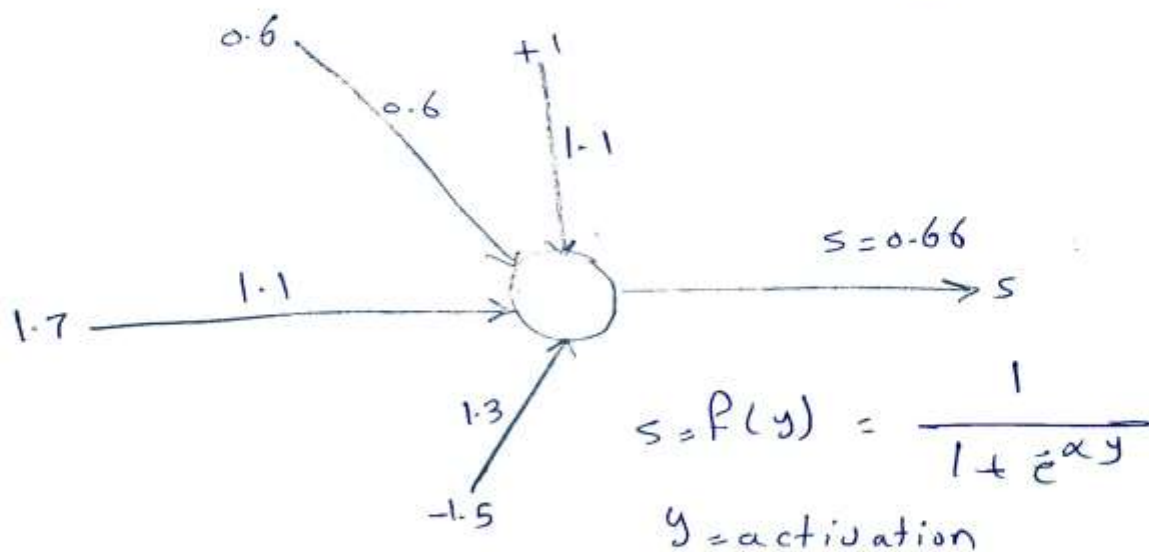


$$y = (0.5)(0.7) + (1.5)(0.9) + (-1.1)(1.2) + (1.3)(1) = 1.68$$

مع استقبل نيورو ~ مدخلات 0.6 ، 1.7 ، -1.5 weights
 0.6 ، 1.1 ، 1.3 على الترتيب ، و يستخدم (sigmoidal P_n)
 على الصورة $P(y) = \frac{1}{1 + e^{-\alpha y}}$ حيث

$y \rightarrow$ activation ، $\alpha \rightarrow$ +ve Parameter

أرجو قيمة α بحيث إشارة الخارج تصبح 0.66 ،
 اعتبر 1.1 = bias weight .



Activation ,

$$y = (0.6)(0.6) + (1.7)(1.1) + (-1.5)(1.3) + (1.1) = 1.38$$

$$\alpha = \frac{1}{y} \ln \left[\frac{P(y)}{1 - P(y)} \right] = \frac{1}{y} \ln \left[\frac{s}{1 - s} \right]$$

$$= \frac{1}{1.38} \ln \left(\frac{0.66}{1 - 0.66} \right) = 0.481$$

$$F(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that

$$\frac{dF(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$

$$= \alpha F(x) [1 - F(x)]$$

Proof $F(x) = \frac{1}{1 + e^{-\alpha x}}$

$$\frac{dF(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$

$$= \frac{\alpha}{(1 + e^{-\alpha x})} \cdot \frac{e^{-\alpha x}}{(1 + e^{-\alpha x})}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \cdot \frac{1 + e^{-\alpha x} - 1}{1 + e^{-\alpha x}}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \left[1 - \frac{e^{-\alpha x}}{1 + e^{-\alpha x}} \right]$$

⑥ -

$$\frac{dP(x)}{dx} = \alpha P(x) [1 - P(x)] \quad \neq$$

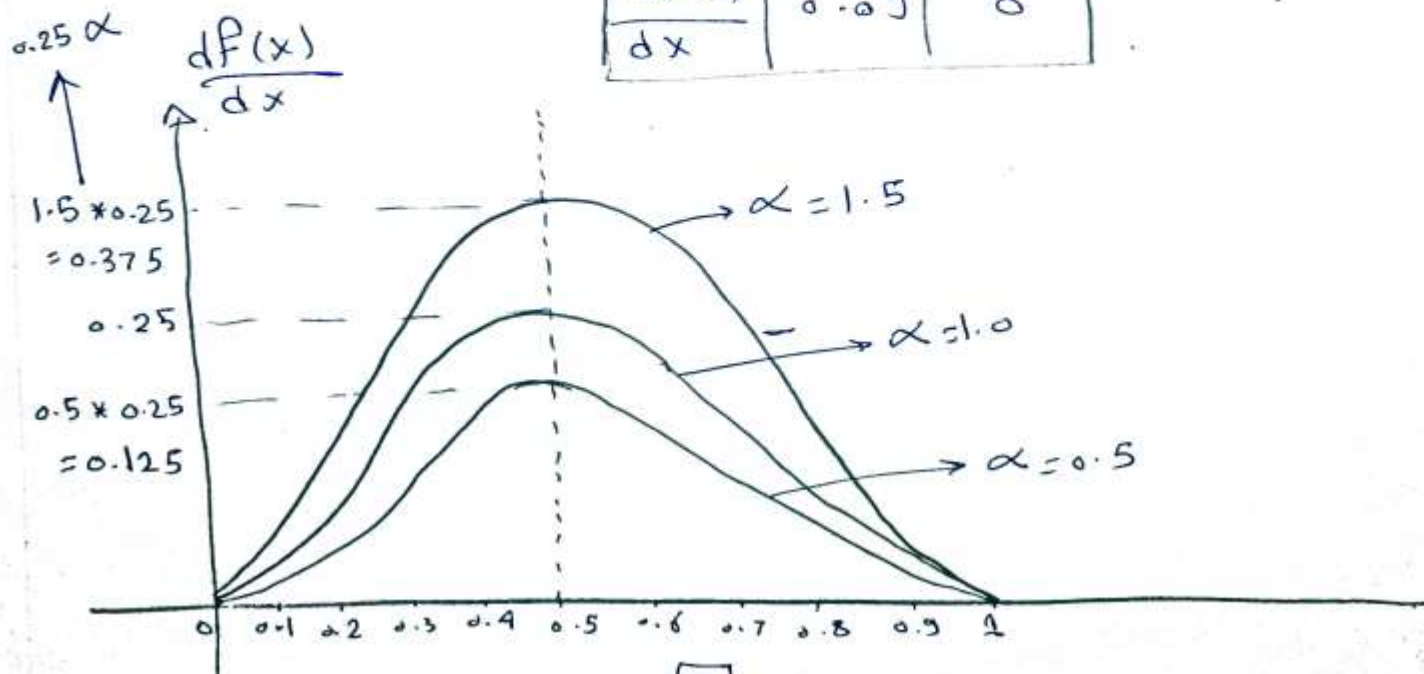
$$\frac{dP(x)}{dx} \quad \text{vs.} \quad P(x) \quad \text{"For } \alpha = 0.5, 1, 1.5$$

له اثبت ان القيمة العظمى للتفاضل $\alpha = 0.25, 0.5, 0.75$ عندما $P(x) = 0.5$

$$\frac{dP(x)}{dx} = \alpha P(x) [1 - P(x)]$$

$P(x)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$\frac{dP(x)}{dx}$	0	0.09				0.25			

$P(x)$	0.9	1
$\frac{dP(x)}{dx}$	0.09	0



→ يتوقع من الشكل السابق أن القيمة العظمى للمشتقة $\frac{dP(x)}{dx}$ هي (0.25α) وذلك عند $P(x) = 0.5$.

لم يمكن إثبات هذه العلاقة رياضيًا عن طريق الآتي.
 يجب النهاية العظمى رياضيًا بأن $\frac{dP(x)}{dx} = 0$

$$\begin{aligned}\frac{dP(x)}{dx} &\propto P(x) [1 - P(x)] \\ &= \alpha P(x) - \alpha P^2(x)\end{aligned}$$

$$\frac{d\left(\frac{dP(x)}{dx}\right)}{dP(x)} = \alpha - 2\alpha P(x) = 0$$

$$P(x) = 0.5$$

$$\left. \frac{dP(x)}{dx} \right|_{\text{Max.}} = \alpha \times 0.5 \cdot [1 - 0.5]$$

$$= 0.25\alpha$$

• BiPolay

نفس الكلام على ال

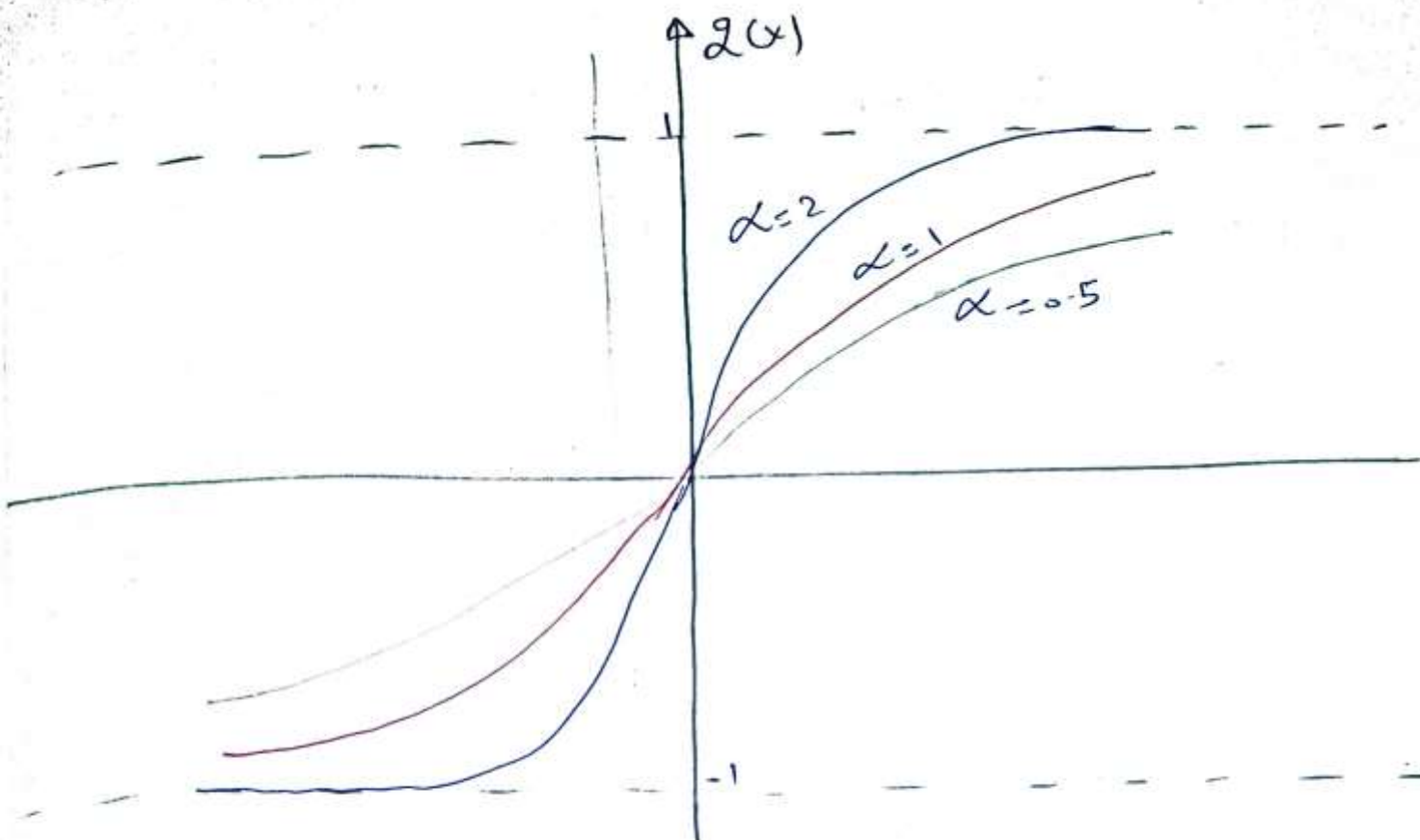
$$g(x) = \frac{2}{1 - e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

Draw, on the same Coordinate axes,
the graphs of $g(x)$ for $\alpha = 0.5, 1, 2$
and comment on these graphs.

verify that $x = \frac{1}{\alpha} \ln \left[\frac{1 + g(x)}{1 - g(x)} \right]$

and, $\frac{dg(x)}{dx} = 0.5 \alpha [1 - g^2(x)]$

x	$g(x), \alpha = 0.5$	$g(x), \alpha = 1$	$g(x), \alpha = 2$
-5	-0.848	-0.987	-0.999
-4	-0.762	-0.964	-0.998
⋮			
0	0	0	0
⋮			
4	0.762	0.964	0.998
5	0.848	0.987	0.999



مع زيادة قيم α فإن الشكل البياني يقترب أكثر وأكثر من المحاور الرأسية وفي النهاية عندما $\alpha \rightarrow \infty$ فإن الدالة تصبح (Bipolar threshold Function)

$$g(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$g(x) + e^{-\alpha x} \quad g(x) = 1 - e^{-\alpha x}$$

$$e^{-\alpha x} = \frac{1 - g(x)}{1 + g(x)}$$

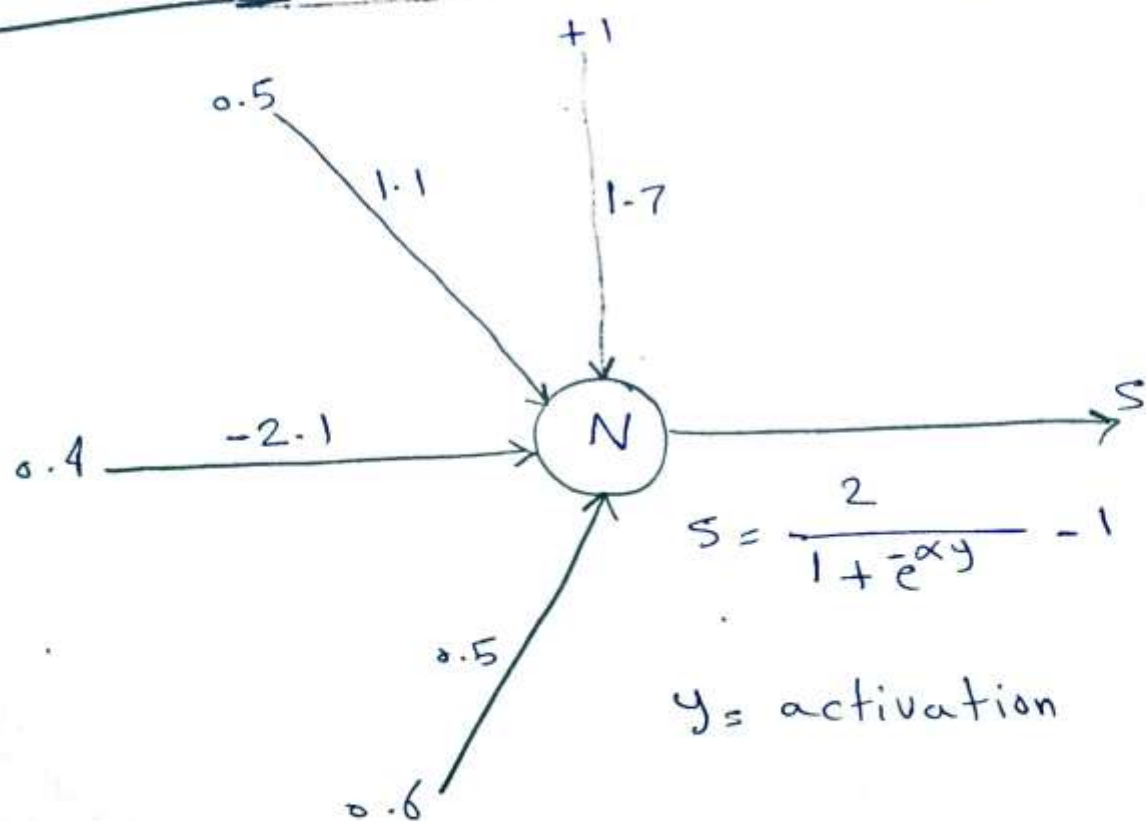
$$-\alpha x = \ln \frac{1 - g(x)}{1 + g(x)}$$

$$x = \frac{1}{\alpha} \ln \left[\frac{1 + g(x)}{1 - g(x)} \right]$$

$$g(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$\frac{dg(x)}{dx} = \frac{(1 + e^{-\alpha x})(\alpha e^{-\alpha x}) - (1 - e^{-\alpha x})(-\alpha e^{-\alpha x})}{(1 + e^{-\alpha x})^2}$$

$$= 0.5 \alpha \left[1 - \left(\frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}} \right)^2 \right] = 0.5 \alpha [1 - g^2(x)]$$



Find the value of the parameter α such
that $S = 0.75$

Activation,

$$y = (0.5)(1.1) + (0.4)(-2.1)(0.6)(0.5) + 1.7 = 1.71$$

$$y = \frac{1}{\alpha} \ln\left(\frac{1+S}{1-S}\right) \quad S = f(y)$$

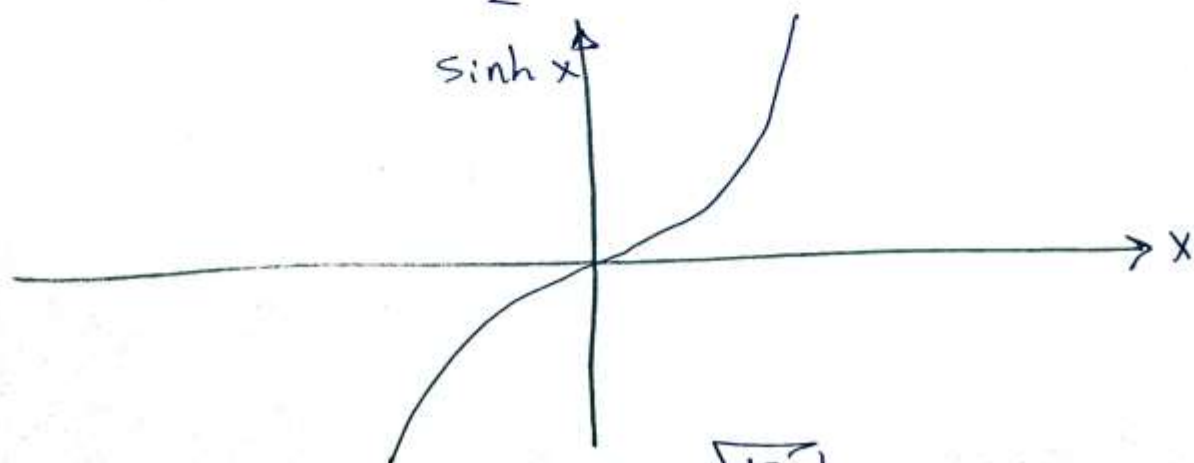
$$\alpha = \frac{1}{y} \ln\left(\frac{1+S}{1-S}\right)$$

$$\underline{S = 0.75}$$

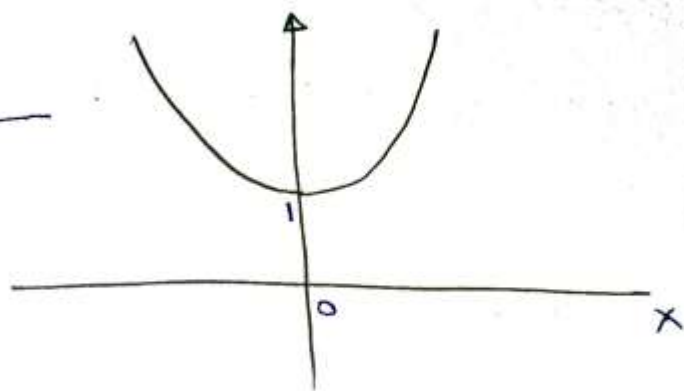
$$\alpha = \frac{1}{1.71} \ln\left(\frac{1+0.75}{1-0.75}\right) = 1.138$$

* Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

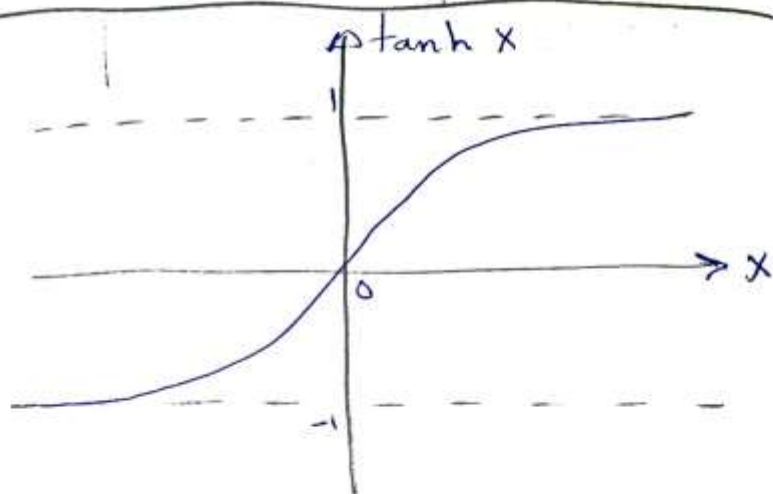


$$\cosh x = \frac{e^x + e^{-x}}{2}$$



$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$= \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$= \frac{2}{1 + e^{-2x}} - 1$$

← المعادلة دي هي بالظبط معادلة [Bipolar sigmoidal Function]

لكن فيه $\alpha = 2$

Generalization, $\tanh x =$

$$\frac{2}{1 + e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$= \frac{e^{\frac{\alpha x}{2}} - e^{-\frac{\alpha x}{2}}}{e^{\frac{\alpha x}{2}} + e^{-\frac{\alpha x}{2}}} = \boxed{\tanh\left(\frac{\alpha x}{2}\right)}$$

($\tanh x$) $\alpha = 2$ gives x